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The plane dynamic problem of coupled thermoelasticity☆

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ABSTRACT

A method of solving two-dimensional inner and outer boundary-value problems of coupled thermoelasticity, taking into account the finite propagation velocity of heat pulses, is proposed, based on constructed fundamental solutions of the corresponding equations. An estimate is given of the coupling of thermomechanical fields in these problems, and the hyperbolic and parabolic models of thermal conductivity are compared. It is shown that the effect of the finite propagation velocity of heat is unimportant even for very short periods of the duration of the processes (comparable with the relaxation time of the heat flux).

The first solution of the dynamic problem of thermal shock at the boundary of a half-space was proposed by Danilovsaya.[1] In recent years, a scientific area has developed in thermomechanics, in which the propagation velocity of heat pulses are taken into account.[2] The existence of experimental facts, which are contrary to Fourier representations, are the reasons for constructing a number of models of heat conduction, which predict a finite propagation velocity of heat waves. We can distinguish a class of models, leading to a hyperbolic heat-conduction equation. Thus, the generalized heat-conduction equation (but ignoring coupling) is used to describe temperature fields, which arise in highly intense heat exchange in pulsed and laser devices, in the laser processing of metals, in plasma deposition, in the high-power channels of nuclear reactors and in many other industrial technological processes.

Despite the large number of investigations in thermoelasticity, it is still an urgent matter to develop analytic and numerical methods of solving boundary-value problems in this area. Below we propose an approach to solving two-dimensional inner and outer boundary-value problems (taking into account the finite propagation velocity of thermal perturbations), which rest on fundamental solutions of the two-dimensional equations of coupled thermoelasticity,[3] with subsequent use of the techniques of singular integral equations.

1. Formulation of the problem

We will consider, in a Cartesian system of coordinates Ox_1x_2, an elastic plate, weakened by a finite number of openings with a common boundary $\Gamma = \cup\Gamma_\nu = \emptyset$, $\nu = 1,2,\ldots,N$. On Γ we specify a stress vector, and also a heat flux, which vary harmonically with time. The problem consists of determining the coupled thermoelastic fields for harmonic or pulsed excitation of the plate, taking into account the finite propagation velocity of heat pulses.

The differential equations of coupled thermoelasticity have the form[2,4]

$$\nabla^2 u_j + \sigma\partial_j e - \frac{3\lambda + 2\mu}{\mu}\alpha_T\partial_j\theta + \frac{1}{\mu}F_j = \frac{\rho}{\mu}\ddot{u}_j, \quad j = 1, 2$$

$$\nabla^2\theta - \frac{1}{\upsilon_T^2}\ddot{\theta} - \frac{1}{a^2}\dot{\theta} - m(\dot{e} + \tau^*\ddot{e}) = -\frac{W}{\lambda_T} - \frac{\tau^*}{\lambda_T}\dot{W} \tag{1.1}$$

$$\nabla^2 = \partial_1^2 + \partial_2^2, \quad \partial_j = \frac{\partial}{\partial x_j}, \quad \sigma = \frac{1}{1-2\nu}, \quad m = \frac{(3\lambda + 2\mu)T_0\alpha_T}{\lambda_T} \tag{1.2}$$

where μ_j are the components of the displacement vector, $e = \partial_l u_l$ ($l = 1, 2$) is the volume expansion, λ and μ are the Lamé constants, ν is Poisson's ratio, α_T is the temperature coefficient of linear expansion of an isotropic body, $\theta = T - T_0$ is the temperature increment (T_0 is

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Table 1

Material	$\lambda \times 10^{-10}$, N/m^2	$\mu \times 10^{-10}$, N/m^2	$\rho \times 10^{-3}$, kg/m^3	$c_\varepsilon \times 10^{-2}$, J/(kg k)	$\alpha_T \times 10^6$, K^{-1}	δ at $T_0 = 330$ K
Non-organic glass	2.48	2.9	2.5	8.33	9	0.0027
Polyvinylacetals	0.393	0.098	1.07	10.8	230	0.487

the temperature of the body in the undeformed and unstressed state and T is the absolute temperature of points of the body), F_j are the components of the intensity of the bulk load, ρ is the density, $\upsilon_T = \sqrt{a^2/\tau^*}$ is the propagation velocity of heat pulses, τ^* is the heat flux relaxation time, $a^2 = \lambda_T/c_\varepsilon\rho$ is the thermal diffusivity, c_ε is the volume heat capacity for constant strain, λ_T is the thermal conductivity of the material and W is the heat-source density function.

The expressions for the components of the stress tensor have the form[5]

$$\sigma_{kj} = \lambda(\partial_l u_l)\delta_{kj} + \mu(\partial_k u_j + \partial_j u_k) - (3\lambda + 2\mu)\alpha_T\theta\delta_{kj} \tag{1.3}$$

where δ_{kj} is the Kronecker delta.

Eqs. (1.1) and (1.2) are coupled. They describe the deformation of a body which occurs due to the action of transient thermal and mechanical actions, and also the inverse effect, namely, the change in the temperature of the body due to deformation.

The coupled part of the heat-conduction Eq. (1.2) has been analysed, and was rewritten as follows[6] (for convenience for $\tau^* = 0$ and $W = 0$):

$$\frac{\partial\Theta(x_1', x_2', \tau)}{\partial\tau} + \delta\dot{e}(x_1', x_2', \tau) = \nabla^2\Theta(x_1', x_2', \tau), \quad \tau > 0$$

where the following new (dimensionless) variables have been introduced

$$\tau = \frac{\upsilon_T^2}{a^2}t, \quad x_j' = \frac{\upsilon_T}{a^2}x_j, \quad j = 1, 2, \quad \Theta(x_1', x_2', \tau) = \frac{3\lambda + 2\mu}{\lambda + 2\mu}\alpha_T\theta(x_1', x_2', \tau),$$

$$\delta = \frac{(3\lambda + 2\mu)^2\alpha_T^2 T_0}{c_\varepsilon\rho(\lambda + 2\mu)}$$

The connectivity parameter δ is fairly small for non-organic and a number of organic glasses and metals (see Table 1), and the contribution of the deformation to the temperature field is unimportant. For polyvinylacetals (polyvinylformal, polyvinylethylal and polyvinylbutyral) the coupling factor is of the order of unity, which indicates a possible increment of the temperature and the stresses when the term reflecting the mechanical coupling is taken into account.

2. The matrix of fundamental solutions of the system

Bearing in mind the harmonic form of the change of the field quantities with time, we put

$$u_j = \mathrm{Re}\lfloor\exp(-i\omega t)U_j\rfloor, \quad \sigma_{kj} = \mathrm{Re}\lfloor\exp(-i\omega t)S_{kj}\rfloor, \quad \theta = \mathrm{Re}[\exp(-i\omega t)U_3]$$

$$e = \mathrm{Re}[\exp(-i\omega t)e_*], \quad F_j = \mathrm{Re}[\exp(-i\omega t)X_j], \quad W = \mathrm{Re}[\exp(-i\omega t)Q]; \; k, j = 1, 2$$

where U_j, S_{kj}, U_3, e_*, X_j, Q are the amplitudes of the corresponding quantities and ω is the angular frequency.

Eliminating the time factor $\exp(-i\omega t)$ in the equations of motion (1.1) and heat conduction (1.2), we can represent them in amplitudes

$$(\nabla^2 + \gamma_2^2)U_j + \sigma\partial_j e_* - \alpha_0\partial_j U_3 = -\frac{1}{\mu}X_j, \quad j = 1, 2$$

$$m\omega(1 + \tau^*\omega)e_* + (\nabla^2 + \gamma_T^2 + i\gamma^2)U_3 = -\frac{1}{\lambda_T}(1 - i\omega\tau^*)Q$$

$$\gamma_2 = \frac{\omega}{c_2}, \quad e_* = \partial_l U_l, \quad l = 1, 2, \quad \alpha_0 = \frac{3\lambda + 2\mu}{\mu}\alpha_T, \quad \gamma_T = \frac{\omega}{\upsilon_T}, \quad \gamma = \sqrt{\frac{\omega}{a^2}} \tag{2.1}$$

Suppose a concentrated force $\mathrm{Re}[(P_1, P_2)\exp(-i\omega t)]$ or a concentrated heat source $\mathrm{Re}[P_3 \exp(-i\omega t)]$, varying harmonically with time, acts at the point (x_{10}, x_{20}). Differentiating the first equation of system (2.1) with respect to the coordinate x_1 and differentiating the second equation with respect to the coordinate x_2, and then adding them, we arrive at the system

$$l_{11}e_* - l_{12}U_3 = -\frac{1}{\mu(1 + \sigma)}(\partial_1 P_1 + \partial_2 P_2)\delta(x), \quad l_{21}e_* + l_{22}U_3 = -\frac{1 - i\omega\tau^*}{\lambda_T}P_3\delta(x)$$

$$l_{11} = \nabla^2 + \gamma_1^2, \quad l_{12} = \frac{\alpha_0}{1 + \sigma}\nabla^2, \quad l_{21} = m\omega(i + \omega\tau^*), \quad l_{22} = \nabla^2 + \gamma_T^2 + i\gamma^2; \quad \gamma_1 = \frac{\omega}{c_1} \tag{2.2}$$

where $\delta(x)$ is the delta function.

Henceforth it is more convenient to integrate system (2.2) in the space $D'(R^2)$ of generalized functions.[7]
We will consider in more detail the case when $P_1 \neq 0$, $P_2 = P_3 = 0$. Introducing the resolving function $\Phi(x_1, x_2)$ using the formulae

$$e_* = l_{22}\Phi, \quad U_3 = -l_{21}\Phi$$

we convert system (2.2) to a fourth-order inhomogeneous differential equation

$$(\nabla^2\nabla^2 + d\nabla^2 + b)\Phi = -\frac{P_1}{\mu(1+\sigma)}\partial_1\delta(x)$$

$$d = \gamma_1^2 + i\gamma^2 + \gamma_T^2 + \frac{\mu\alpha_0\beta_0}{1+\sigma}(i + \omega\tau^*), \quad b = \gamma_1^2(i\gamma^2 + \gamma_T^2), \quad \beta_0 = \alpha_0\frac{T_0}{\lambda_T}\omega$$

We will represent the solution of this equation in the form

$$\Phi = \frac{iP_1}{4\mu(1+\sigma)(\mu_2^2 - \mu_1^2)}\sum_{j=1}^{2}(-1)^{j-1}\partial_1 H_0^{(1)}(\mu_j r), \quad r = \sqrt{(x_1 - x_{10})^2 + (x_2 - x_{20})^2} \tag{2.3}$$

where $H_p^{(1)}(x)$ is the Hankel function of the first kind of order p, and μ_j ($\mathrm{Im}\mu_j > 0, j = 1, 2$) is the root of the equation $z^4 - dz^2 + b = 0$.
Taking representation (2.3) into account, we conclude that system (2.1) can be split into three independent equations

$$(\nabla^2 + \gamma_2^2)U_l = \frac{P_1 a_0}{4i\mu}\sum_{j=1}^{2}(-1)^j d_j\partial_1\partial_l H_0^{(1)}(\mu_j r) - \delta_{1l}\frac{P_1}{\mu}\delta(x), \quad l = 1, 2$$

$$U_3 = \frac{P_1 a_0 m\omega(i + \omega\tau^*)}{4i\mu}\sum_{j=1}^{2}(-1)^j\partial_1 H_0^{(1)}(\mu_j r) \tag{2.4}$$

Here

$$a_0 = \frac{1}{(1+\sigma)(\mu_1^2 - \mu_2^2)}, \quad d_j = (\sigma+1)d - \gamma_2^2 - i\gamma^2 - \gamma_T^2 - \sigma\mu_j^2, \quad j = 1, 2$$

We introduce the following notation

$$g_l^{(k)} = -\delta_{lk}H_0^{(1)}(\gamma_2 r) + a_0\sum_{j=0}^{2}(-1)^j d_j b_j\partial_l\partial_k H_0^{(1)}(\mu_j r), \quad l, k = 1, 2$$

$$g_l^{(3)} = \frac{1+\nu}{1-\nu}\alpha_T\mu\beta\sum_{j=1}^{2}(-1)^j\partial_l H_0^{(1)}(\mu_j r), \quad g_3^{(k)} = m\omega(i + \omega\tau^*)a_0\sum_{j=1}^{2}(-1)^j\partial_k H_0^{(1)}(\mu_j r)$$

$$g_3^{(3)} = \mu\beta\sum_{j=1}^{2}(-1)^j(\gamma_1^2 - \mu_j^2)H_0^{(1)}(\mu_j r)$$

$$d_0 = 1, \quad b_0 = -\frac{1}{a_0\gamma_2^2}, \quad b_j = \frac{1}{\gamma_2^2 - \mu_j^2}, \quad j = 1, 2; \quad \beta = -\frac{1 - i\omega\tau^*}{\lambda_T(\mu_1^2 - \mu_2^2)} \tag{2.5}$$

Integrating Eq. (2.4) in the space of generalized functions D′, we obtain

$$U_n = \frac{P_1}{4i\mu}g_n^{(1)}, \quad n = 1, 2, 3$$

Considering the cases $P_2 \neq 0$, $P_1 = P_3 = 0$ and $P_3 \neq 0$, $P_1 = P_2 = 0$ in the same way, we obtain final expressions for the amplitudes of the displacements and the temperature

$$U_n^{(m)} = \frac{P_m}{4i\mu}g_n^{(m)}, \quad n, m = 1, 2, 3 \tag{2.6}$$

The matrix of the fundamental solutions $||g_n^{(m)}||$ is used later when constructing integral representations of the solutions of the boundary-value problem.

3. The integral equations of the boundary-value problem of coupled thermoelasticity

The complex boundary equalities on the contour Γ in the case of force and thermal excitation have the form

$$S_1 - e^{2i\psi}S_2 = 2e^{i\psi}(X_{1n} - iX_{2n}), \quad S_1 - e^{-2i\psi}\tilde{S}_2 = 2e^{-i\psi}(X_{1n} + iX_{2n}), \quad -\lambda_T\frac{\partial U_3}{\partial n} = \Phi$$

$$S_1 = S_{11} + S_{22}, \quad S_2 = S_{22} - S_{11} + 2iS_{12}, \quad \tilde{S}_2 = S_{22} - S_{11} - 2iS_{12} \tag{3.1}$$

where X_{1n} and X_{2n} are the components of the external load vector acting on the boundary area with normal n, Φ is the heat flux specified on the contour Γ and ψ is the angle between the outward normal to the contour Γ and the Ox_1 axis.

To determine the wave fields of the stresses and the temperature in the plate, we will introduce integral representations of the amplitudes of the displacements and the temperature in the form of convolutions

$$U_n(z) = \int_\Gamma \sum_{m=1}^{3} Z_m(\zeta)g_n^{(m)}(\zeta, z)ds, \quad \zeta \in \Gamma = \bigcup_{\nu=1}^{N}\Gamma_\nu, \quad n = 1, 2, 3;$$

$$Z_m(\zeta) = \{Z_{m\nu}(\zeta), \zeta \in \Gamma_\nu\} \tag{3.2}$$

where $Z_m(\zeta)$ are unknown "densities", and $g_n^{(m)}$ are the components of the matrix of the fundamental solutions (2.6) defined by formulae (2.5). Using expressions (3.2) and the law (1.3) we obtain the components of the stress tensor, after which we determine the combinations of the field quantities occurring in condition (3.1). Substituting the limiting values of these quantities into the boundary conditions we obtain a system of singular integral equations of the second kind on the contour Γ

$$\pm 4iW_p(\zeta_0) + \int_\Gamma \sum_{m=1}^{3} W_m(\zeta)K_{pm}(\zeta, \zeta_0)ds = F_p(\zeta_0), \ \zeta_0 \in \Gamma = \bigcup_{\nu=1}^{N}\Gamma_\nu, \quad p = 1, 2, 3 \tag{3.3}$$

Here

$$W_{1,2} = \frac{1}{2}e^{\mp i\psi}(Z_1 \pm iZ_2), \quad W_3 = \mu\alpha_T Z_3$$

$$K_{ll}(\zeta, \zeta_0) = -\gamma_2 H_1^{(1)}(\gamma_2 r_0)\exp\lfloor(-1)^{l-1}i(\alpha_0 + \psi - 2\psi_0)\rfloor -$$
$$-a_0\sum_{j=1}^{2}(-1)^j\mu_j\{\sigma(d_jb_j\mu_j^2 + \alpha^*)\exp[(-1)^{l-1}i(\psi - \alpha_0)] +$$
$$+d_jb_j\mu_j^2\exp[(-1)^{l-1}i(\alpha_0 + \psi - 2\psi_0)]\}H_1^{(1)}(\mu_j r_0), \quad l = 1, 2$$

$$K_{lk}(\zeta, \zeta_0) = a_0\exp[(-1)^k i(3\alpha_0 - 2\psi_0 - \psi)]\sum_{j=0}^{2}(-1)^j d_jb_j\mu_j^3 H_3^*(\mu_j r_0) -$$
$$-a_0\sigma\exp[(-1)^{l-1}i(\alpha_0 - \psi)]\sum_{j=1}^{2}(-1)^j\mu_j(d_jb_j\mu_j^2 + \alpha^*)H_1^{(1)}(\mu_j r_0), \quad k = 1, 2; \ l + k = 3$$

$$K_{l3}(\zeta, \zeta_0) = \beta\frac{1+\nu}{1-\nu}\sum_{j=1}^{2}(-1)^j\{\exp[(-1)^{l-1}2i(\alpha_0 - \psi_0)]\mu_j^2 H_2^{(1)}(\mu_j r_0) - (\gamma_2^2 - \mu_j^2)H_0^{(1)}(\mu_j r_0)\}$$

$$K_{3l}(\zeta, \zeta_0) = iE\sigma T_0\alpha_T^2\omega\alpha_0\sum_{j=1}^{2}(-1)^j\mu_j^2\{\exp[(-1)^{l-1}i(\psi + \psi_0 - 2\alpha_0)]H_2^{(1)}(\mu_j r_0) -$$
$$-\exp[(-1)^{l-1}i(\psi - \psi_0)]H_0^{(1)}(\mu_j r_0)\}$$

$$K_{33}(\zeta, \zeta_0) = -\frac{2}{\mu_1^2 - \mu_2^2}\sum_{j=1}^{2}(-1)^j\mu_j(\gamma_1^2 - \mu_j^2)H_1^{(1)}(\mu_j r_0)\cos(\alpha_0 - \psi_0)$$

$$\alpha^* = 2(1+\nu)\alpha_T m\omega(i + \omega\tau^*), \quad H_3^*(\mu_j r_0) = \frac{16i}{\pi\mu_j^3 r_0^3} + H_3^{(1)}(\mu_j r_0), \quad \alpha_0 = \arg(\zeta - \zeta_0)$$

$$F_{1,2}(\zeta_0) = \frac{1}{\mu}(N \pm iT)(\zeta_0), \quad F_3(\zeta_0) = -\frac{2\alpha_T}{1 - i\omega\tau^*}\Phi(\zeta_0)$$

N and T are the amplitudes of the normal and shear forces, applied to the contour Γ, and E is Young's modulus; the upper sign in Eq. (3.3) corresponds to the inner problem and the lower sign corresponds to the outer problem.

The normal circumferential stress $\sigma_{\theta\theta}$ and the temperature θ on the contour Γ are given by the formulae

$$\sigma_{\theta\theta}^{\pm} = \mathrm{Re}\lfloor \exp(-i\omega t)S_{\theta\theta}^{\pm}\rfloor, \quad \theta = \mathrm{Re}[\exp(-i\omega t)U_3]$$

$$S_{\theta\theta}^{\pm}(\zeta_0) = 2\mu\sigma\left\{\pm\frac{i}{(1-\nu)\sigma}[W_1(\zeta_0)+W_2(\zeta_0)] + \int_{\Gamma}\sum_{m=1}^{3} W_m(\zeta)R_m(\zeta,\zeta_0)ds\right\} - N$$

$$U_3(\zeta_0) = \int_{\Gamma}\sum_{m=1}^{3} W_m(\zeta)R_3^{(m)}(\zeta,\zeta_0)ds$$

$$R_l(\zeta,\zeta_0) = -a_0\exp[(-1)^l i(\alpha_0-\psi)]\sum_{j=1}^{2}(-1)^j\mu_j(d_jb_j\mu_j^2+\alpha^*)H_0^{(1)}(\mu_j r_0)$$

$$R_3^{(l)}(\zeta,\zeta_0) = m\omega(i+\omega\tau^*)a_0\exp[(-1)^l i(\alpha_0-\psi)]\sum_{j=1}^{2}(-1)^j\mu_j H_1^{(1)}(\mu_j r_0); \quad l = 1, 2$$

$$R_3(\zeta,\zeta_0) = -\frac{\beta(1+\nu)}{\sigma(1-\nu)}\sum_{j=1}^{2}(-1)^j(\gamma_2^2-\mu_j^2)H_0^{(1)}(\mu_j r_0)$$

$$R_3^{(3)}(\zeta,\zeta_0) = \frac{\beta}{\alpha_T}\sum_{j=1}^{2}(-1)^j(\gamma_1^2-\mu_j^2)H_0^{(1)}(\mu_j r_0) \tag{3.4}$$

4. Pulsed excitation of a plate with a hole

Suppose a trapezoidal pulse with respect to time

$$\frac{N}{N_0} = \begin{cases} t/t_b, & 0 \le t \le t_b \\ 1, & t_b \le t \le t_c \\ (t-t_d)/(t_c-t_d), & t_c \le t \le t_d \\ 0, & t_d \le t < \infty \end{cases}$$

acts on the contour of the hole Γ, where N_0, t_b, t_c and t_d are the parameters of the pulse.

In this case the solution of the dynamic problem can be "chosen" from a packet of monochromatic waves using a unilateral Fourier integral transformation in time

$$F_*(x_1,x_2,\omega) = \frac{1}{\sqrt{2\pi}}\int_0^{\infty}F(x_1,x_2,t)e^{i\omega t}dt, \quad F(x_1,x_2,t) = \sqrt{\frac{2}{\pi}}\mathrm{Re}\int_0^{\infty}F_*(x_1,x_2,\omega)e^{-i\omega t}d\omega$$

Applying a Fourier transformation to the equations of motion and heat conduction (1.1) and (1.2), we obtain system (2.1) with respect to the corresponding transformants.

The field quantities are found using an inverse Fourier transformation.

5. Results of calculations

Suppose an unbounded elastic medium is weakened by a single hole. To obtain the constructed algorithm numerically we will introduce parametrization of the contour of the hole Γ

$$\zeta = \zeta(\varphi_k), \quad \zeta_0 = \zeta(\varphi_{0l}), \quad 0 \le \varphi_k, \varphi_{0l} \le 2\pi$$

Table 2

Material	$E\times 10^{-10}$, N/m^2	c_ε, J/(kg K)	$\rho\times 10^{-3}$, kg/m^3	ν	$\alpha_T\times 10^6$, K^{-1}	α_T, J/(m s k)	δ
Aluminium	7	861	2.7	0.34	26	207	0.038
Polystyrene	0.255	1077	1.04	0.3	70	0.16	0.015
Polyvinyl-butyral	0.275	1077	1.07	0.4	230	0.16	0.431

Applying the method of mechanical quadratures[8] to the singular integral equations (3.3) and bearing in mind the interpolation formula (for an odd number of points of subdivision of the contour)

$$W(\varphi_{0l}) = \frac{1}{n}\sum_{k=1}^{n}(-1)^{k+l}W(\varphi_k)\operatorname{cosec}\frac{\varphi_k-\varphi_{0l}}{2},\quad \varphi_k = \frac{\pi(2k-1)}{n},\quad \varphi_{0l} = \frac{2\pi(l-1)}{n},$$
$$l = 1, 2, \ldots, n$$

we reduce them to a system of a linear algebraic equations in the values of the densities $W_m(\varphi_k)$ at the interpolation points φ_k

$$\sum_{k=1}^{n}\sum_{m=1}^{3}W_m(\varphi_k)\left[\pm 4i(-1)^{k+l}\delta_{pm}\operatorname{cosec}\frac{\varphi_k-\varphi_{0l}}{2}+2\pi K_{pm}(\varphi_k,\varphi_{0l})\frac{ds}{d\varphi}(\varphi_k)\right] = nF_p(\varphi_{0l})$$
$$p = 1, 2, 3;\quad l = 1, 2, \ldots, n$$

$$\frac{ds}{d\varphi}(\varphi_k) = \sqrt{[\xi'(\varphi_k)]^2+[\eta'(\varphi_k)]^2},\ \zeta = \xi+i\eta$$

Relations (3.4) can be converted to the form

$$S^{\pm}_{\theta\theta}(\varphi_{0l}) = \frac{2\pi\sigma}{n}\sum_{k=1}^{n}\left\{\pm\frac{i}{(1-\nu)\sigma}(-1)^{k+l}\operatorname{cosec}\frac{\varphi_k-\varphi_{0l}}{2}[W_1(\varphi_k)+W_2(\varphi_k)]+\right.$$
$$\left.+2\pi\sum_{m=1}^{3}W_m(\varphi_k)R_m(\varphi_k,\varphi_{0l})\frac{ds}{d\varphi}(\varphi_k)\right\},\quad l = 1, 2, \ldots, n$$

$$U_3(\varphi_{0l}) = \frac{2\pi}{n}\sum_{k=1}^{n}\sum_{m=1}^{3}W_m(\varphi_k)R_3^{(m)}(\varphi_k,\varphi_{0l})\frac{ds}{d\varphi}(\varphi_k)$$

In the dynamic problem we used the inverse Fourier transformation algorithm, based on an approximation of the transform using continued fractions.[9,10]

Graphs of the amplitude of the modulus of the normal circumferential stress $|S_{\theta\theta}|$ against the relative wave number γ_{2r} (r = 0.1 m) are shown in Fig. 1 for an unbounded plate with a hole, and in Fig. 2 we show the same graphs for a plate of finite dimensions. The parametric equations of the contours of the holes in Fig. 1 and the contours of the finite plates in Fig. 2 have the following form

$$\text{a) } \zeta = \mathrm{R}e^{i\varphi},\ \text{b) } \zeta = R_1\cos\varphi+iR_2\sin\varphi,\ \text{c) } \zeta = R(e^{i\varphi}+0.14036e^{-3i\varphi}),$$

$$\text{d) } \zeta = R(e^{i\varphi}+0.25e^{-2i\varphi}).$$

Here

$$R = 10^{-1}\,\mathrm{m},\ R_1 = 1.3\cdot 10^{-1}\,\mathrm{m},\ R_2 = 0.7\cdot 10^{-1}\,\mathrm{m},\ 0\le\varphi\le 2\pi$$

A normal force N = 1 N/m^2 and zero heat flux Φ are specified on the contour Γ. We took polyvinylbutyral (coupling factor δ = 0.431) as the material, the physical/chemical characteristics of which[2,5] at T_0 = 293 K are given in the last row of Table 2. The continuous curves are drawn taking into account the coupling of the deformation and temperature fields, while the dashed curves are the results obtained ignoring coupling.

In Fig. 3 we show the results of calculations of the time dependence of $\dot{\alpha} = \sigma_{\theta\theta}/N_0$ when the plate is excited by a trapezoidal pulse of pressure N_0 = 1 N/m^2 and duration $t_d = 10^{-6}$ s (for $t_b = 0.1\times 10^{-6}$ s and $t_c = 0.9\times 10^{-6}$ s). The parametric equations of the contours of the holes are the same as for versions *b* and *d* in Figs. 1 and 2. The calculations were carried out for polyvinylbutyral (relaxation time of the heat flux $\tau^* = 10^{-5}$ s). The continuous curves correspond to the hyperbolic model of heat conduction (taking into account the relaxation of the heat flux), while the dashed curves correspond to the parabolic model (ignoring relaxation).

We can conclude from the results obtained that for such materials as polyvinylbutyral, the coupling effect of connectivity may be considerable, particularly in the region of peak values of the excitation frequency. The amplitude-frequency characteristics of finite plates are also changed considerably.

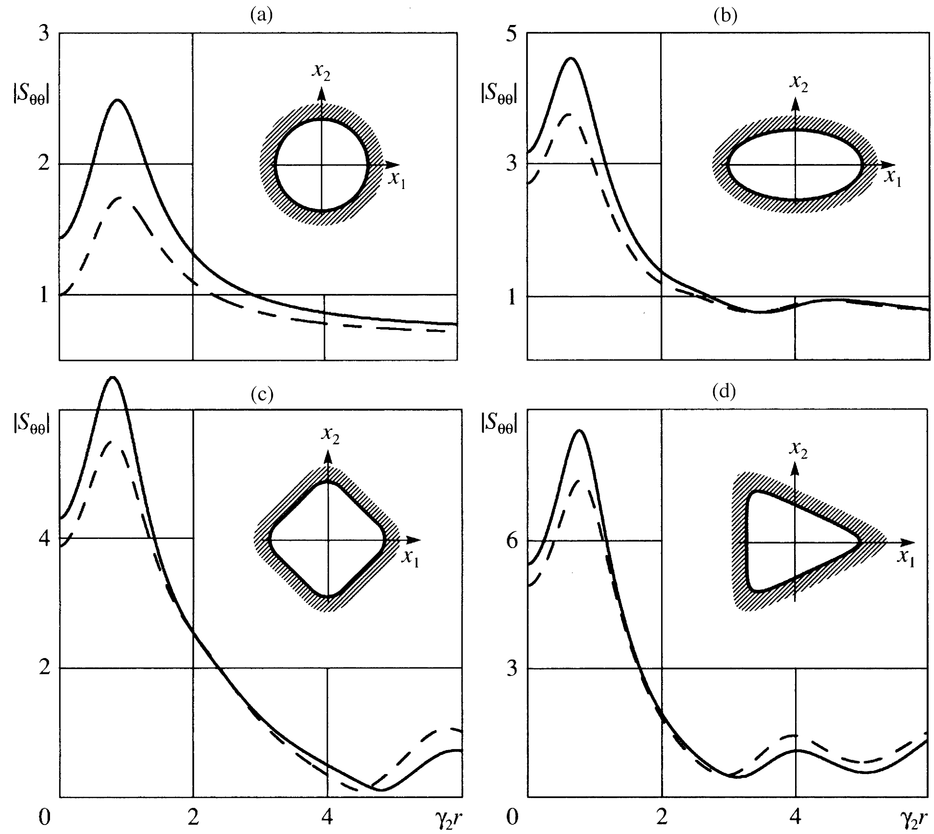
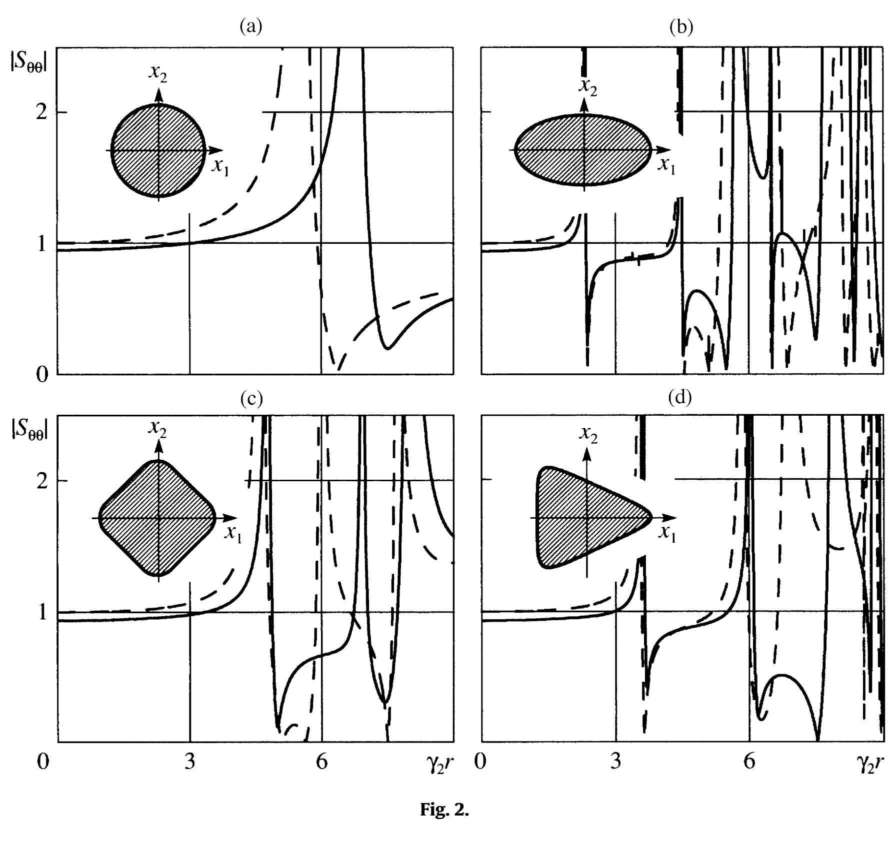

(a)
(b)
(c)
(d)
$|S_{\theta\theta}|$
x_1
x_2
$\gamma_2 r$

Fig. 1.


(a)
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$|S_{\theta\theta}|$
x_1
x_2
$\gamma_2 r$

Fig. 2.

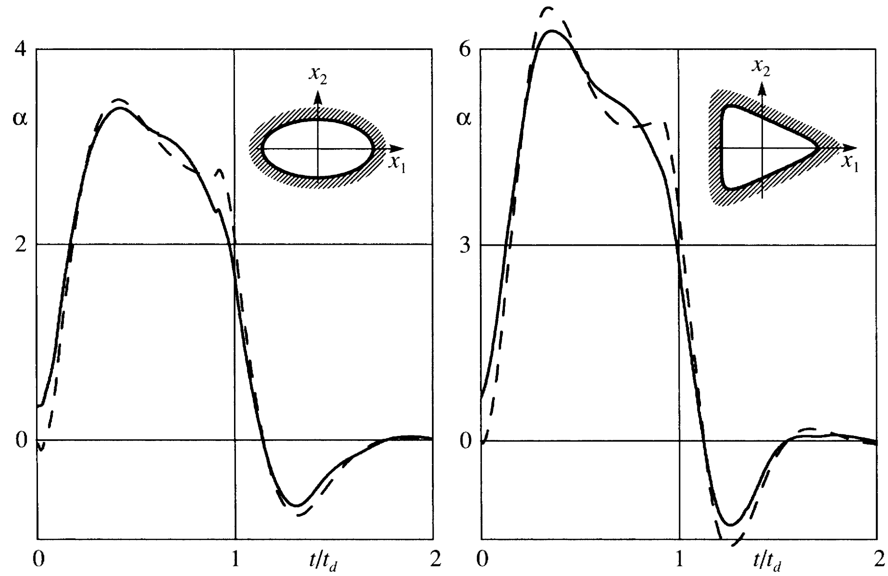

Fig. 3.

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